Unit -1
Fundamental Aspects of Vibrations

Oscillating Motions:

- The study of vibrations is concerned with the oscillating motion of elastic bodies and the force associated with them.
- All bodies possessing mass and elasticity are capable of vibrations.
- Most engineering machines and structures experience vibrations to some degree and their design generally requires consideration of their oscillatory motions.
- Oscillatory systems can be broadly characterized as linear or nonlinear.

Linear systems:

The principle of superposition holds. Mathematical technique available for their analysis are well developed.

Nonlinear systems:

The principle of superposition doesn't hold the technique for the analysis of the nonlinear systems are under development (or less well known) and difficult to apply. All systems tend to become nonlinear with increasing amplitudes of oscillations. There are two general classes of vibrations– free and forced.

FREE VIBRATIONS

Objective of the present section will be to write the equation of motion of a system and evaluate its natural frequency, which is mainly a function of *mass, stiffness,* and *damping* of the system from its general solution.

- In many practical situation, damping has little influence on the natural frequency and may be neglected in its calculation.
- In absence of damping, the system can be considered as conservative and principle of conservation of energy offers another approach to the calculation of the natural frequency.
- The effect of damping is mainly evident in diminishing of the vibration amplitude at or near the resonance.Free *vibration* takes place when a system oscillates under the action of forces inherent in the system itself due to initial disturbance, and when the externally applied forces are absent. The system under free vibration will vibrate at *one or more* of its *natural frequencies*, which are *properties of the dynamical system*, established by its mass and stiffness distribution.

Forced Vibration: -

- The vibration that takes place under the excitation of external forces is called forced vibration.

- If excitation is harmonic, the system is forced to vibrate at *excitation frequency.* If the frequency of excitation coincides with one of the natural frequencies of the system, a condition of *resonance* is encountered and dangerously large oscillations may result, which results in failure of major structures, i.e., bridges, buildings, or airplane wings etc.
- Thus, calculation of natural frequencies is of major importance in the study of vibrations.
- Because of friction & other resistances vibrating systems are subjected to *damping* to some degree due to dissipation of energy.
- Damping has very *little effect on natural frequency* of the system, and hence the calculations for natural frequencies are generally made based on no damping.
- Damping is of great importance in *limiting the amplitude* of oscillation at resonance.

Degree of freedom: -

The number of independent co-ordinates required to describe the motion of a system is termed as degrees of freedom.

For example,
Particle - 3 dof (positions)
rigid body-6 dof (3-positions and 3-orientations)
Continuous elastic body - infinite dof (three positions *to each particle of the body).*

- If part of such continuous elastic bodies may be assumed to be rigid (or lumped) and the system may be considered to be dynamically equivalent to one having finite dof (or lumped mass systems).
- Large number of vibration problems can be analyzed with sufficient accuracy by reducing the system

Linear and Non-Linear Vibrations: When the vibrations are represented by linear differential equations and laws of superposition are applicable for the system, we have linear systems. Nonlinearvibrations are experienced when large amplitudes are encountered and laws of superposition are not applicable.

Longitudinal, Transverse and Torsional Vibrations: When the motion of mass of the system is parallel to the axis of the system, we have longitudinal vibrations. When the motion of mass is perpendicular to the system axis the vibrations are Transverse vibrations and when the mass twists and untwists about the axis the vibrations are Torsional vibrations. Up and down motion of mass in a spring mass system represents longitudinal vibrations. Vibration of a cantilever beam represents Transverse vibrations. The twisting and untwisting of a disc attached at the end of a shaft represents Torsional vibrations.

Vector representation of SHM: Any SHM can be represented as by the equation, $x = A\sin\omega t$ where x is the displacement , A is the amplitude , ω is the circular frequency and t is the time. Differentiating eqn.1 w.r.t. 't' we have velocity vector and differentiating eqn 1 twice we have

2) Repeat the above problem given, x1 = 2Cos (ωt + 0.5) and x2 = 5Sin(ωt + 1.0) . The angles are in radians. (Hint: In the above problem, the angles are to be converted to degrees. Ans. A = 6.195, θ = 73.49 ˚)

3) Add the following harmonic motions analytically or graphically.
x1 = 10 Cos (ωt + π/4) and x2 = 8 Sin(ωt + π/6) .

4) A body is subjected to 2 harmonic motions

x1 = 15sin (ωt + π/6) , x2 = 8 cos(ωt + π/6), what harmonic is to be given to the body to it to equilibrium.

Solution:

Let the harmonic to be given to the two harmonics to make it to be in equilibrium be Asin (ωt + ϕ)

Therefore, Asin (ωt + ϕ)+ x1 + x2 = 0

Hence, Asinωtcosϕ+Acosωt sin ϕ+15sinωt cosπ/6+15cosωt sinπ/6+8 cosωtcosπ/6+8 sinωt sinπ/6 = 0

sinωt (A cos ϕ+8.99038)+ cosωt(A sin ϕ+14.4282) = 0

Therefore, A cos ϕ = -8.99038

A sin ϕ= - 14.4282

Therefore, tan ϕ = A sin ϕ / A cosϕ = 14.4282/8.99038 , ϕ = 58.062˚

From, A cos ϕ = -8.99038, substituting for ϕ = 58.062˚, A = 17.00

Therefore, the motion is x= 17sin (ωt + 58.062˚)

Beats Phenomenon: Consider two harmonics x1 and x2 of slightly different frequencies and the A cosϕ resulting motion will not be a SHM. Due to existence of different frequencies the phase difference of the two vectors keeps on changing and shifting w.r.t time. The two harmonics when in phase have their resultant amplitude to be sum of individual amplitudes and when they are out of phase the resultant amplitude is difference of individual amplitudes. This phenomenon of varying of resultant amplitude is called as Beats and this occurs at a frequency given by the difference of the individual frequencies of the two vectors.

Fourier Theorem: Any periodic motion can be represented in terms of sine and cosine terms called as Fourier series. The process of obtaining the Fourier series of a periodic motion is called Harmonic analysis, i.e.

F(t) a periodic function can be represented as the acceleration vector. If x1 and x2 are two displacement vectors with same frequencies then the phase difference between them is given by ϕ.

Principle of Superposition: When two SHM of same frequencies are added the resulting motion is also a harmonic motion. Consider two harmonic motions $x1 = A1 \sin \omega t$ and $x2 = A2 \sin(\omega t + \phi)$. Then if x is the resultant displacement, $x = x1 + x2$. The resultant amplitude $x = A \sin(\omega t + \theta)$, where A is the resultant amplitude and is acting at an angle θ w.r.t vector x1.

The above addition of SHMs can also be done graphically.

Sample Problems: (1) Add the following harmonics analytically and check the solution graphically

$x1 = 3\sin(\omega t + 30^\circ)$, $x2 = 4\cos(\omega t + 10^\circ)$

Solution:

Given: $x1 = 3\sin(\omega t + 30^\circ)$, $x2 = 4\cos(\omega t + 10^\circ)$

Analytical method:

We know that, $x = x1 + x2 = A \sin(\omega t + \theta)$

Make x1 and x2 to have same Sin terms always, i.e., $x2 = 4\cos(\omega t + 10^\circ + 90^\circ) = 4\sin(\omega t + 100^\circ)$

Hence, $A \sin(\omega t + \theta) = 3\sin(\omega t + 30^\circ) + 4\sin(\omega t + 100^\circ)$

Expanding LHS and RHS

$A\sin\omega t \cos\theta + A\cos\omega t \sin\theta = 3\sin\omega t \cos 30^\circ + 3\cos\omega t \sin 30^\circ + 4\sin\omega t \cos 100^\circ + 4\sin\omega t \sin 100^\circ$

$A\sin\omega t \cos\theta + A\cos\omega t \sin\theta = \sin\omega t(1.094) + \cos\omega t(5.44)$

Comparing the coefficients of $A\cos\theta$ and $A\sin\theta$ in the above equation

$A\cos\theta = 1.094$, $A\sin\theta = 5.44$, $\tan\theta = A\sin\theta / A\cos\theta = 5.44/1.904$

Therefore, $\theta = 70.7^\circ$ and $A = 1.094/\cos 70.7^\circ = 5.76$.

Graphical Method: Draw ox the reference line. With respect to ox, draw oa equal to 3 units in length at an angle of 30° to ox and ob equal to 4 units at an angle of 100° to ox. Complete the vector polygon by drawing lines parallel to oa and ob to intersect at point c. Measure oc which should be equal to A and the angle oc makes with ox will be equal to θ. All angles measured in anticlockwise direction.

$F(t) = a0 + a1\cos\omega t + a2\cos 2\omega t + a3\cos 3\omega t + \ldots\ldots.an \cos n\omega t$

$+ b1\sin\omega t + b2\sin 2\omega t + b3\sin 3\omega t + \ldots\ldots.. bn\sin n\omega t$

The constants a0,a1,a2,,,,,and b1,b2,b3.... etc., are obtained using the following formulae:

$a0 = (\omega/2\pi) \int F(t)$, in the limits 0 to $2\pi/\omega$

$an=(\omega/\pi) \int F(t)\cos(n\omega t)\, dt$, in the limits 0 to $2\pi/\omega$

$bn= =(\omega/\pi) \int F(t)\sin(n\omega t)\, dt$, in the limits 0 to $2\pi/\omega$

Oscillatory Motion

- Repeat itself regularly for example pendulum of a wall clock
- Display irregularity for example earthquake

Periodic Motion – This motion repeats at equal interval of time *T*.

Period of Oscillatory – The time taken for one repetition is called period.

Frequency - $f = \frac{1}{T}$, It is defined reciprocal of time period.

The condition of the periodic motion is

$$x(t+T) = x(t)$$

Where motion is designated by time function *x*(*t*).

Harmonic motion:

- Simplest form of periodic motion is harmonic motion and it is called simple harmonic motion (SHM). It can be expressed as

  $$x = A\sin 2\pi\frac{t}{T}$$

  Where *A* is the amplitude of motion, *t* is the time instant and *T* is the period of motion.

- Harmonic motion is often represented by projection on line of a point that is moving on a circle at constant speed. (SHM Animation)

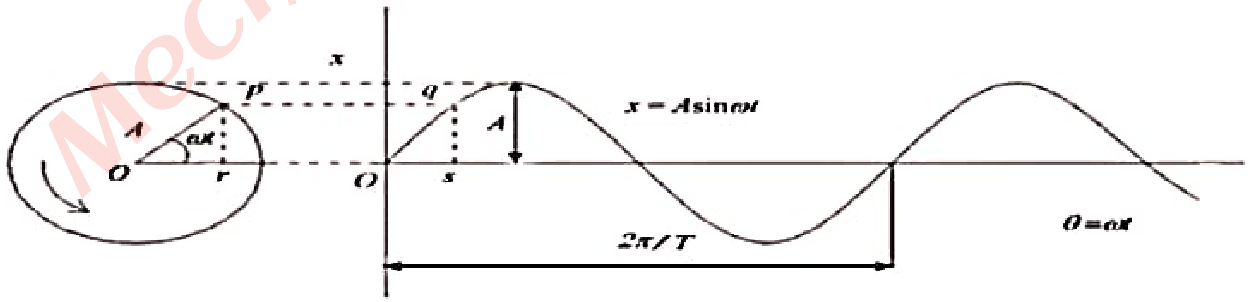

Figure 1.1: The Simple Harmonic Motion

Example 1.1: A harmonic motion has an amplitude of 0.20 cm and a period of 0.15 sec. determine the maximum velocity and acceleration.

Solution: The frequency is given as

$$\omega = \frac{2\pi}{T} = \frac{2\pi}{0.15} = 41.89 \text{ rad/s}$$

The maximum velocity is given as

$$(\dot{x})_{max} = \omega A = 41.89 \times 0.2 = 8.38 \text{ cm/s}$$

The maximum acceleration is given as

$$(\ddot{x})_{max} = \omega^2 A = (41.89)^2 \times 0.2 = 350.94 \text{ cm}^2\text{/s}$$

Equation of Motion (EOM):